Lesson 12. Dynamic Programming – Review

- Recall from Lessons 5-11:
  - A **dynamic program** models situations where decisions are made in a sequential process in order to optimize some objective
  - **Stages** $t = 1, 2, \dots, T$
    - stage $T \leftrightarrow$ end of decision process
  - **States** $n = 0, 1, \dots, N \leftarrow$ possible conditions of the system at each stage
  - Two representations: **shortest/longest path** and **recursive**

Shortest/longest path		Recursive
node t_n	$\leftrightarrow$	state n at stage t
edge $(t_n, (t+1)_m)$	$\leftrightarrow$	allowable decision x_t in state n at stage t that results in being in state m at stage $t+1$
length of edge $(t_n, (t+1)_m)$	$\leftrightarrow$	contribution of decision x_t in state n at stage t that results in being in state m at stage $t+1$
length of shortest/longest path from node t_n to end node	$\leftrightarrow$	value-to-go function $f_t(n)$
length of edges (T_n, end)	$\leftrightarrow$	boundary conditions $f_T(n)$
shortest or longest path	$\leftrightarrow$	recursion is min or max: $f_t(n) = \min_{x_t \text{ allowable}} \text{ or } \max \left\{ \begin{pmatrix} \text{contribution of} \\ \text{decision } x_t \end{pmatrix} + f_{t+1} \begin{pmatrix} \text{new state} \\ \text{resulting} \\ \text{from } x_t \end{pmatrix} \right\}$
source node 1_n	$\leftrightarrow$	desired value-to-go function value $f_1(n)$

Example 1. Simplexville Oil needs to build capacity to refine 1,000 barrels of oil and 2,000 barrels of gasoline per day. Simplexville can build a refinery at 2 locations. The cost of building a refinery is as follows:

Oil capacity per day	Gas capacity per day	Building cost ($ millions)
0	0	0
1000	0	5
0	1000	7
1000	1000	14

The problem is to determine how much capacity should be built at each location in order to minimize the total building cost. Assume that the capacity requirements must be met exactly.

a. Formulate this problem as a dynamic program by giving its shortest path representation.
b. Formulate this problem as a dynamic program by giving its recursive representation. Solve the dynamic program.